

# Eight circle theorems page

(a pdf version of <http://www.timdevereux.co.uk/maths/geompages/8theorem.php>)

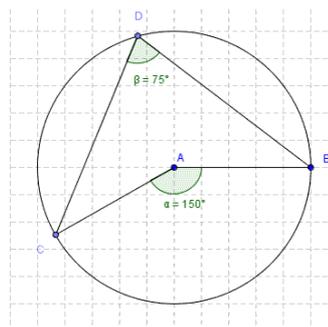
The Eight Theorems:

- |                        |   |   |
|------------------------|---|---|
| First circle theorem   | - | angles at the centre and at the circumference.  |
| Second circle theorem  | - | angle in a semicircle.                          |
| Third circle theorem   | - | angles in the same segment.                     |
| Fourth circle theorem  | - | angles in a cyclic quadrilateral.               |
| Fifth circle theorem   | - | length of tangents.                             |
| Sixth circle theorem   | - | angle between circle tangent and radius.        |
| Seventh circle theorem | - | alternate segment theorem.                      |
| Eighth circle theorem  | - | perpendicular from the centre bisects the chord |

Circle Theorem 1:            Angles at the centre and at the circumference

The angle at the centre is twice the angle at the circumference.

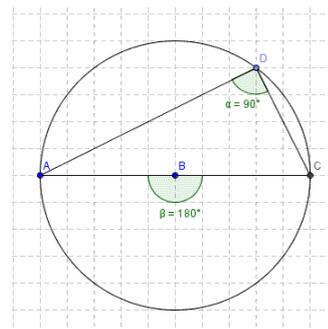
(Note that both angles are facing the same piece of arc, CB)



Circle Theorem 2:            Angle in a semicircle

The angle in a semi-circle is 90°.

(This is a special case of theorem 1, with a centre angle of 180°.)

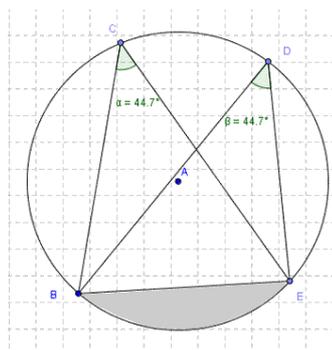


Circle Theorem 3:            Angles in the same segment

Angles in the same segment are equal.

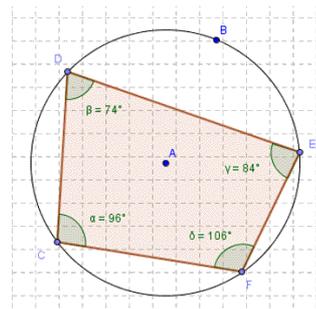
(The two angles are both in the major segment;

I've coloured the minor segment grey)



Circle Theorem 4:            Angles in a cyclic quadrilateral

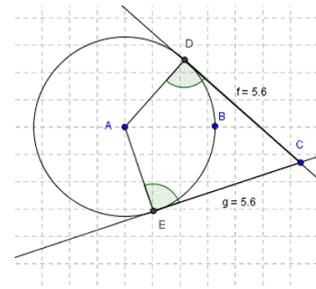
Opposite angles in a cyclic quadrilateral add up to 180°. [A cyclic quadrilateral has all 4 vertices (corners) touching a circle]



Circle Theorem 5: Length of tangents

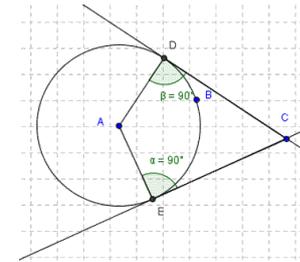
The lengths of the two tangents from a point to a circle are equal.

$CD = CE$



Circle Theorem 6: Angle between circle tangent and radius

The angle between a tangent (eg DC) and a radius (eg AD) in a circle is  $90^\circ$ .

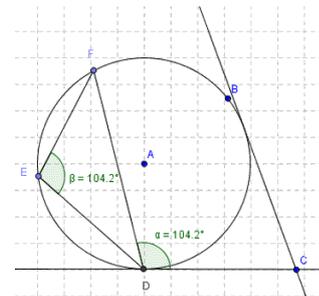


Circle Theorem 7: Alternate segment theorem

The angle ( $\alpha$ ) between the tangent (DC) and the chord (DF) at the point of contact (D) is equal to the angle ( $\beta$ ) in the alternate segment\*. ie  $\alpha = \beta$

[This is a weird theorem, and needs a bit more explanation: Chord DF splits the circle into two segments. In one segment, there is an angle,  $\beta$ , 'facing' the chord, DF – this segment is called the alternate segment. Partly in the other segment, and partly outside the circle altogether, the angle  $\alpha$ , is between the chord DF and the tangent DC]

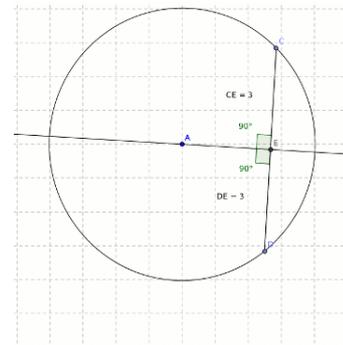
(\*Thank you, BBC Bitesize, for providing me with wording for this theorem!)



Circle Theorem 8: Perpendicular from the centre bisects the chord

The perpendicular from centre A cuts the chord CD at E, the centre point of the chord, so that

$DE = EC$



The Dynamic Geometry pages (starting at <http://www.timdevereux.co.uk/maths/geompages/index.php>) are much more fun than mere pictures & text – have a look!

Tim Devereux, 15/08/2013